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Design of comprehensive income products for retirement using utility functions

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Abstract
An approach for designing a menu of comprehensive income products for retirement (CIPRs) is proposed and demonstrated. The approach entails four steps: defining and characterising member types based on selected attributes; specifying a utility function to capture the objectives and preferences for each member type; conducting analysis of candidate investment and drawdown strategies, and hence select a product design; and communication to members. The last step uses attributes to describe the type of investor for which a product is designed, as well as setting out the key product features and the outcomes it may deliver.

JEL Classification: **D14, D15, E21, G11, G23**

Keywords
CIPRs, drawdown strategies, portfolio choice, product design, retirement savings, utility

1. Introduction

The design of retirement products is a challenge facing the pension industry around the world. This article addresses this challenge within an Australian setting. It aims to provide direction to superannuation funds on how they might develop a menu of 'comprehensive income products for retirement' (CIPRs) under the Australian Government's proposed retirement incomes framework.[1] These products – which will be referred to as 'MyRetirement' products in this article – are envisaged as providing a balance of income, risk management (e.g. longevity risk) and flexibility (e.g. some access to funds). An approach is presented for developing a menu of MyRetirement products for a range of member types, with utility functions playing a central role in capturing differing objectives and preferences. Utility functions provide the 'scoring mechanism' for the range of outcomes delivered by a particular investment and drawdown strategy, thus allowing 'optimal'

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strategies to be identified or candidate strategies to be compared. This article draws on Warren (2019), who addresses utility-based analysis of investment strategies, and argues that utility functions should be purposefully selected to represent particular investors. This concept is extended here to an investment organisation catering for a variety of investor types. This article also builds on and extends the Members Default Utility Function (MDUF) initiative.[2]

The proposed approach comprises four steps. First is defining and characterising the member types for which a set of products will be designed. Second is specifying a utility function to represent each member type. Third is undertaking analysis to select the product to offer each member type. This could commence by identifying the 'optimal' strategy that maximises expected utility, before examining other strategies. Product selection might be guided by weighing the magnitude of utility loss from adopting a sub-optimal strategy against considerations such as feasibility, cost of delivery and likely acceptance by members. Metrics are used to characterise the type of outcomes that various strategies deliver. The final step is devising a plan for communicating products to members.

The use of attributes to define and characterise member types is fundamental to the approach. Member types are described by the outcomes they are concerned about (e.g. income, bequests or their balance), their risk tolerance and attributes that capture their personal situation such as household status or homeownership. These attributes are embedded in the utility function used to evaluate the outcomes, and provide a foundation for communication and engagement with members. Utility functions are not mentioned when communicating.

The approach envisages that a superannuation fund might offer a MyRetirement product to a retiring member based on what is known about them. The offer would be communicated by providing information on the type of member for which the product is designed, the key features of the product and the type of outcomes that it can be expected to deliver. A description of other available products is provided to allow the member to gauge if another option might be more suitable for their needs. Products should be presented in a way that members can understand and adverse behavioural influences are minimised. The approach aims to match investors with suitable products, and hence dovetails with the design and distribution obligations that were legislated in 2019.[3]

The approach is demonstrated through developing a mock menu of Australian MyRetirement product options for six hypothetical member types. The analysis makes a number of simplifications, noting that the purpose is to illustrate the approach rather than put forward products that might be used in practice. The demonstration provides guidance on how the approach would be applied.

This article is written for practitioners, but connects to a number of strands in the academic literature. Analysis of saving and consumption strategies over multiple periods within an expected utility framework stems back to the development of life cycle models, with Merton (1969, 1971) and Samuelson (1969) providing seminal contributions. The specific concern here is the retirement or decumulation phase, where the aim is to formulate a joint strategy for investment and drawdown under uncertainty over asset returns and mortality. This issue is addressed in an Australian context by authors such as Hulley et al. (2013), Iskhakov et al. (2015), Andréasson and Shevchenko (2017), Andréasson et al. (2017), Butt et al. (2019) and others. This body of research highlights the strong influence of the means-tested Age Pension on retirement strategies in an Australian context, which is acknowledged here by incorporating the Age Pension in the illustrative analysis.

This article also relates to research into the design of retirement savings defaults. MyRetirement products are envisaged as a 'soft' default to be offered for consideration. Nevertheless, many members may accept the option presented as defaults are influential, for example, see Choi et al. (2005) and Chetty et al. (2014). This places onus on the provider to ensure that the product offered is appropriate for the member, and that other available options are communicated effectively. The proposed approach is motivated by two observations from the literature. The first is the considerable heterogeneity across retired members: Spicer et al. (2016) and Asher et al. (2017) provide a sense for this diversity. Further, inappropriate defaults can lead to meaningful welfare losses where heterogeneity is substantial (Alserda et al., 2019; Butt et al., 2018; Dahlquist et al., 2018). Member segmentation and the range of products offered thus require careful consideration. Second is that members can make choices that might not be in their own best interests for behavioural reasons: see Benartzi and Thaler (2007) for a review. Notable influences include poor financial literacy (Agnew et al., 2013; Bateman et al., 2012; Lusardi and Mitchell, 2014) and sensitivity to how information is presented (e.g. Agnew and Szykman, 2005; Bateman et al., 2014; Card and Ransom, 2011). This underlines the importance of designing and then communicating defaults to mitigate the potential for adverse behavioural effects. These considerations point towards the need to be careful in defining member types, tailoring utility functions to capture differing objectives and preferences and communicating products. This article provides guidance on all these matters.

Section 2 commences by discussing the use of utility functions in designing retirement products. Section 3 outlines the proposed approach, which is demonstrated in Section 4 through Section 6. Section 7 concludes.

2. Utility functions and the design of retirement products

Utility-based analysis can support the design of retirement products by operating as a 'scoring mechanism' for evaluating the full range of outcomes stemming from investment and drawdown strategies. The process involves forming a distribution of potential outcomes arising from a strategy, using a utility function to place scores on all outcomes and then aggregating to generate an overall expected utility score. In the retirement context, the outcomes of concern typically entail a stream of income (or consumption). However, some members may be concerned with the retirement account balance itself, as a bequest, a funding source for possible expenditures (e.g. moving into an aged care facility, precautionary funds for medical expenses), or simply because they are wealth focused. Utility functions can be specified to cover this range of potential objectives.

Utility-based analysis offers three advantages when designing a menu of retirement products. First is the flexibility to tailor for investors with differing objectives and preferences. Second is the capacity to evaluate outcomes that span multiple periods, as utility scores may be added up over time. This accommodates trading-off outcomes that are spread over time, which is particularly important where income streams are the main focus. Third, utility-based analysis may be used to either identify an 'optimal' strategy that maximises expected utility or compare candidate strategies. The illustrative analysis presented in this article showcases these advantages.

The alternative is evaluating strategies using metrics that summarise certain elements of the distribution of outcomes, such as expected income, income percentiles, value-at-risk or conditional value-at-risk and the age at which the retirement balance is exhausted (failure rates). The superiority of utility-based measures over mean–variance techniques and metrics is argued by Adler and Kritzman (2007), Goetzmann et al. (2007), Butt and Khemka (2015) and Warren (2019). Utility functions support evaluation of *all* potential outcomes and can be rigorously connected to investor objectives and preferences. Metrics require trading-off measures that only partially summarise the distribution of outcomes, and can be problematic when complex trade-offs are involved. For instance, utility functions evaluate income streams and the residual balance at death as a bequest by condensing the trade-offs into a single score. A metric-based approach might try to solve this problem by generating measures such as expected income, income shortfall and perhaps the probability or expected value of a bequest. It is unclear how this complex series of trade-offs spanning multiple objectives and time periods might be evaluated. Nevertheless, the approach envisages using utility and metrics in conjunction, in the spirit of multi-criteria decision-making methods (see Velasquez and Hester, 2013). Utility-based analysis acts as the primary 'engine' for evaluating strategies, while metrics provide the 'dashboard' by characterising outcomes from strategies. Metrics also provide a reasonableness test through making the outcomes visible.

There are two main hurdles in using utility functions. First, they need to be specified, and can be tricky to parameterise. Warren (2019) discusses this issue and suggests some methods. In specifying utility functions for six member types, this article shows that sensible representations of member objectives and preferences are possible. The second hurdle is that utility functions are difficult to communicate, often not being well understood. A way of structuring communications is proposed here that helps to overcome this hurdle. Utility functions are also (validly) criticised for being subjective. However, this criticism can be equally levied at metric-based methods that require choosing metrics and how they are traded off. Subjectivity seems inescapable either way.

3. Approach for designing a menu of retirement products

This section details the four steps in the approach, which include (a) defining and characterising the member types for which products will be designed; (b) specifying a tailored utility function for each member type; (c) conducting analysis to identify and evaluate candidate investment and drawdown strategies, and hence select a suitable product design; and (d) devising a communication plan.

3.1. Defining and characterising member types

Defining and characterising the member types for which products are to be developed is a pivotal first step. This is a matter of customer segmentation and product range determination, rather than an investment problem. It entails considering the attributes by which members are characterised, degree of diversity among members, access to information about members, organisational capabilities, competitive positioning, regulation and so on. There will be trade-offs between the number of products, cost, feasibility and suitability.

The attributes used to describe members and segment the member base are fundamental. In the context of an Australian retiree, the following attributes can be highly influential in determining an appropriate strategy:

- Retirement account balance – Account balance dictates capacity to generate income, in conjunction with any other assets outside of the retirement account. It also affects Age Pension eligibility under the means testing rules.
- Homeownership – Homeownership impacts the income required during retirement. Those owning a home will typically require lower income than those who rent. Homeownership does not impact Age Pension eligibility, and might be taken into account when setting the strength of any bequest (or precautionary) motive with respect to the retirement account. The importance of housing to decisions in retirement in an Australian setting is established by Cho and Sane (2013), Ding (2014) and Andréasson et al. (2017).
- Excess other assets – Ideally assets other than the retirement account and any home need to be accounted for as a further component of the resources available to satisfy needs during retirement. Excess other assets might be considered as net of debt, and exclusive of any amounts retained for precautionary or transactional purposes.[4] The potential significance of other assets is indicated by Spicer et al. (2016) and Daley et al. (2018: 104).

- Household status – Couples can be distinguished from single-person households in various ways, including income required during retirement; the value of any Age Pension; potential existence of dual retirement accounts; the need to model longevity to the last surviving member; and possible links to a bequest motive. Evidence that behaviour can differ significantly between couples and single-person households is provided by Spicer et al. (2016), Andréasson et al. (2017) and Asher et al. (2017).

Characterising member types also requires nominating the outcomes of concern and preferences over those outcomes. These items feed directly into utility function specification, which is addressed in the next sub-section.

- Outcomes of concern – The outcomes that the member cares about can dictate the type of utility function. Three main categories are income during retirement, bequests and the value of the retirement fund balance itself. The investment horizon over which outcomes are to be evaluated also needs to be nominated.
- Preferences – Taking a stand on the preferences over differing outcomes for each member type is tantamount to making assumptions about their aspirations and the willingness and capacity to bear risk.

Other attributes might be taken into account, subject to significance and availability of information. Candidates include health status; plans for major expenditure such as aged care; scope to earn additional income; existence of dependants; and, whether family or other support may be available, for example, well-off relatives. Scope needs to be set so that the analysis remains meaningful yet manageable.

3.2. Specifying utility functions

Warren (2019) addresses the selection and parameterisation of utility functions, focusing on power utility and reference-dependent utility. This sub-section builds on this work, expanding to discuss minimum acceptable outcomes, treatment of bequests and expected utility calculations. Outcomes are described as ‘consumption’ in this sub-section, but could be taken as referring to either income or wealth as befits the circumstance.

Reference-dependent utility functions may be appropriate where there exists a *specific target*, for example, consumption in retirement. They have been typically based on the ‘value function’ component of prospect theory (see Kahneman and Tversky, 1979; Tversky and Kahneman, 1992),[5] and are used to analyse retirement outcomes by Blake et al. (2013) in a UK setting and Butt et al. (2019) and Khemka et al. (2020) in Australia. Power utility may be more appropriate where the concern is to maximise the utility from the *overall level* of an outcome of interest, for example, the consumption that can be attained, given the available balance. Power utility has been the traditional work horse for life cycle models, and is applied in an Australian retirement context by Cho and Sane (2013), Hulley et al. (2013), Iskhakov et al. (2015), Bell et al. (2017a, 2017b) and Butt et al. (2019). Appendix 1 presents equations for reference-dependent and power utility.

Specifying a ‘minimum acceptable outcome’ that the member needs to avoid may be appropriate in some situations. This treatment might be motivated by avoiding poverty (see Suari-Andreu et al., 2019), or risk capacity (see Klements, 2018) in the sense that there may be limited capacity to endure outcomes below some threshold. One method is to incorporate a consumption floor in the utility function (see Appendix 1), which has been applied in an Australian setting by Kingston and Thorp (2005), Ding (2014), Iskhakov et al. (2015), Andréasson et al. (2017) and Andréasson and

Shevchenko (2017). A disadvantage is that utility becomes undefined below the floor. Another technique is to impose a minimum threshold that attracts a very large negative utility score. This allows utility to remain defined below the minimum, with the large negative scores then dominating expected utility. This technique will attach higher utility to strategies that minimise the probability of falling below the minimum as the first priority. Having minimised this probability, strategies are then ranked on utility across the remaining outcomes. The method is amenable to being applied in conjunction with reference-dependent utility. A disadvantage is that the minimum acceptable outcome becomes a knife-edge. By contrast, the consumption floor approach progressively ramps up the penalty as outcomes approach the floor. Another disadvantage is that certainty equivalents can no longer be meaningfully calculated if the minimum is breached.

3.2.1. Bequests, precautionary balances and aged care. Some members may intend to use their available balance for purposes other than generating income to consume. This includes bequests and precautionary balances to cover medical or aged care costs.[6] These elements can act as partial substitutes, as funds set aside might satisfy any of these purposes. Further, all three purposes can interact with homeownership (see Lockwood, 2018; Suari-Andreu et al., 2019). Appendix 1 presents a commonly used equation for utility from bequests and discusses implementation issues.

3.2.2. Parameterisation. Parameterising utility functions is difficult, and inevitability entails an element of judgement. Warren (2019) shows that parameter choice can be guided by analysis of how the utility function will evaluate outcomes to understand the trade-offs that will be made (see Appendix 2). The efficacy of a parameterisation may also be gauged by considering whether the indicated strategies deliver reasonable outcomes. The remainder of this sub-section expands on three matters related to parameterisation that are particularly relevant in a retirement setting.

- Risk or loss aversion – Utility parameters might be set for relatively high risk aversion or loss aversion when analysing retirement strategies for two reasons. First, evidence exists that risk aversion increases at older ages (Brooks et al., 2018; Yao et al., 2011), although there is some debate over the significance of this finding (see Kesavayuth et al., 2020). Second, retirees often rely on income from their retirement account for living expenses, and may be averse to substantial income reductions from a risk capacity perspective (Klements, 2018).
- Curvature – Prospect theory prescribes risk-seeking behaviour in the realms of losses (Tversky and Kahneman, 1992). It is questionable whether this is appropriate in a retirement context. Applying a curvature parameter on losses of less than one is consistent with imposing a proportionately lower penalty on losses as consumption falls further below target. A retirement product provider might take the stance that a curvature parameter on losses of greater than one is more suitable when members rely on drawing an income to live on, and thus could become increasingly destitute as their incomes decline. Any adjustments to the curvature parameter should be accompanied by appropriate adjustments to the weighting (i.e. loss aversion) parameter.[7] However, caution is needed as this treatment is at odds with indications arising from experimental studies. Power utility and utility functions with consumption floors impose an increasing penalty on losses as consumption declines.
- Outcome targets – Reference-dependent utility functions require a reference point to be specified. In a retirement context, the consumption (i.e. income) target is related to the concept of ‘adequacy’. Two main approaches for setting the target are replacement rates and consumption budgets,[8] and a multitude of calculation methods exist (see Chybalski and Marcinkiewicz, 2016). A further consideration is whether any target should be treated as

constant or time-varying through the retirement phase. The choice will depend on the member type, and whether it is appropriate to design the product to deliver increasing, stable or declining income.

3.2.3. Estimating expected utility and certainty equivalents. Expected utility is estimated by aggregating utility scores across the entire distribution of outcomes. It helps to conceptualise a strategy as delivering a range of potential outcome paths, with each path generating a level of total utility. Expected utility is estimated by aggregating the total utilities across these paths, either through averaging or probability weighting. Appendix 1 presents an equation for estimating expected utility. Two issues are whether to allow for a time preference parameter and probability of survival. Both place lower weights on outcomes occurring later in retirement when calculating expected utility. This is readily understood for time preference, which acts as a discount rate. The effect of probability of survival may need further explanation.

Applying a probability of survival to each age amounts to an assumption that the member cares less about outcomes that are delivered later in retirement because there is a lower chance they will be experienced. For example, the Australian Life Tables (2015–2017)[9] indicate that an Australian male retiring at age 65 years has a 99.0% chance of surviving to age 66 years, but only an 8.5% chance of surviving 30 years to age 95 years (mortality improvements aside). This means that the utility generated from consumption at age 66 years receives nearly full weight in calculating expected utility, while consumption at age 95 years receives only an 8.5% weight. Applying such mortality weighting will (roughly) induce a preference for strategies where expected outcomes are pitched towards life expectancy.

Whether it is appropriate to discount outcomes occurring later in retirement is a moot point. A product provider might take the stance that the aim is to cater equally for outcomes across the entire retirement phase. Further, many members may be expecting a product to provide for them if they survive to a very old age, and hence may implicitly consider outcomes later in retirement to be just as important as those occurring earlier. While time preference can be easily excluded with little consequence, discarding the mortality weighting is more problematic. The alternative is to assume survival through to some target age when the member dies. For instance, assuming death occurs at age 95 years would establish an analysis that evaluates strategies on how well they provide for a member retiring at age 65 years who lives a further 30 years. Whether to apply time preference and mortality weighting are open issues, and will depend on what is assumed about member objectives and preferences.

Finally, it may be helpful to convert utility scores into certainty equivalents. Where outcomes are a consumption or income stream, this involves solving for the certainty equivalent consumption (CEC) or certainty equivalent income (CEI) that delivers the same expected utility as the uncertain consumption or income stream arising under the strategy. The calculation method will depend on the situation. In some instances, the utility equation may be rearranged so that CEC or CEI is estimated as a function of the expected utility from the strategy.[10] Another method is to locate the CEC or CEI using iterative search.

3.3. Analysis of strategies and product design

Warren (2019) summarises the main modelling components in detail, which include the following:

- Set of candidate assets
- Simulated investment returns arising from investing in those assets

- An investment strategy
- A drawdown strategy
- A plan for what combination of drawdowns and/or terminal portfolio value is evaluated as outcomes
- Utility function for performing the evaluation
- Other elements that impact the admissible strategies and their outcomes, including Age Pension eligibility rules, tax rates, minimum drawdown rules, any constraints, mortality rates and so on.

A model is built to simulate outcomes conditional on candidate investment and drawdown strategies. Expected utility arises by applying the utility function to these outcomes. This method can be used to identify the ‘optimal’ strategy that delivers the highest expected utility, or to compare candidate strategies based on expected utility.

Various modelling choices need to be made. The most precise method is to identify optimal investment and drawdown strategies using dynamic programming. However, less technically demanding techniques may be more practical. These include proposing and testing pre-determined dynamic rules, or identifying a preferred static strategy at the commencement of retirement. The latter might be done with the intent to review the strategy occasionally: this approach is applied in the illustrative example that appears later. Restricting candidate assets to a few ‘building blocks’ can also simplify both product analysis and implementation. In the context of retirement product design, this might include a growth portfolio, a defensive portfolio and annuities, with the latter potentially including term annuities, life annuities and/or deferred annuities. These building blocks could be sourced as separate investment units that are combined in different weights across the retirement product range.

While utility is useful for comparing strategies, as one overall score it does not reveal the outcomes that can be expected. Metrics play this important role, and are useful for understanding and communicating what a strategy is doing. They also provide a check that a proposed strategy delivers acceptable outcomes. Useful metrics in a retirement context include: expected income and selected income percentiles across time; magnitude of any shortfall versus target; age at which the retirement account balance is exhausted; and expected bequests at each age. Where the retirement balance itself is the outcome of concern, useful metrics may include expected balance and its standard deviation, balance percentiles, drawdowns and shortfall measures.

Finally, selecting a particular strategy involves weighing up a range of considerations, including utility scores, outcomes as revealed by metrics, feasibility and cost of product delivery, the likely acceptance by members and the business and regulatory environment. The ‘optimal’ strategy that maximises expected utility might be used as benchmark for comparison, with a view to understanding the utility loss from adopting alternative strategies.

3.4. Communication

Utility functions are an effective analysis tool but have limited value for communication. Most members – and probably even Trustee Boards – would not understand them. Communication may be based on describing the *attributes* of the member types for which products are designed, the key *features* of those products and the *outcomes* that they might be expected to deliver. Metrics can be used to describe the outcomes and convey risk.

The attributes used to describe member types should be linked to the utility functions and the analysis underpinning the product designs. For example, one product might be described as designed for a single member with a modest balance, owns a house and desires a certain level of target income, but could endure ending up on the Age Pension. Another product might be described as designed for a couple with a high balance who rent, are quite averse to ending up on Age Pension and want to leave a bequest if possible. Another product might be described as designed for members who care about their fund balance, are averse to large losses, and so on. Such descriptions imply particular utility functions without mentioning them directly. The example presented in Section 6 should make this communication mode more tangible.

The approach dovetails with the need to offer CIPRs as an option for consideration, the emerging design and distribution obligations and the fact that providers may not have complete information on members. The initial offer is made as a 'best guess' of what product might be most suitable. This is accompanied by similar descriptions of other available products, including the type of members that they are designed for. The intent is to flag to members where another product might be more appropriate, so they can make further enquiries or seek advice. A key aim is to match investors with suitable products.

The relation with financial advice is worth mentioning. Seeking specific personal advice upon retirement may be preferable to selecting products without assistance, and the proposed approach should help facilitate conversations between financial planners and members. The approach also fills the gap for members that might not seek advice. A further issue is how communications relate to the advice regulations (which were under review at the time of writing).[11] It is also possible that restrictions around advice may be eased to facilitate development of CIPRs under the Retirement Income Covenant that is currently under consideration.[12]

4. Illustrating the approach: defining member types and their utility functions

The approach is now illustrated using a stylised example. This section demonstrates the first two steps by defining six hypothetical member types for which products are to be designed, and then specifying a utility function for each. This provides the setup for the following two sections, which address the third step of analysing strategies and product design and the fourth step of communications for one product. All dollar amounts are in Australian dollars.

4.1. Member attributes

Table 1 lists the six member types along with the proposed names of the products to be designed. The menu includes three default products intended for members where there is limited knowledge apart from their account balance (called 'Income A', 'Income B' and 'Income C'). Three other products will be designed to cater for more rounded member types described by a broader range of attributes, including a household ('Family'), a wealthy member ('Well-off') and a balance-focused member ('Nest-egg'). The latter is concerned with their retirement account balance only, while the other five member types are concerned with income and possibly a bequest. The Nest-egg product caters for members who wish to focus on the balance, perhaps because they plan to access it to support some known expenditure or they have a strong precautionary motive, for example, a member in poor health.

Attributes used to characterise each member type appear in panel A of Table 1. The 'personal' attributes at the top are the main elements used to describe members and include the following:

- Balance at retirement – The three default members span low balance (less than \$300,000), medium balance (\$300,000–\$500,000) and high balance (above \$500,000). The household is assumed to have a balance of between \$200,000 and \$500,000, and the wealthy member,

Table 1. Illustrative MyRetirement product menu – Characteristics of six member types and associated utility functions.

	DEFAULTS			OTHER OPTIONS		
Member type	*Low balance*	*Medium balance*	*High balance*	*Household*	*Wealthy*	*Balance-focused*
MyRetirement product	*Income A*	*Income B*	*Income C*	*Family*	*Well-off*	*Nest-egg*
Panel A: Member attributes						
Personal attributes						
Balance at retirement	Up to $300,000	$300,000 – $500,000	Over $500,000	$200,000 – $500,000	Above $500,000	Any
Home owner	No	No	Yes	Yes	Yes	–
Excess other assets	–	–	–	$<$ $200,000	$>$ $200,000	–
Household status	Individual	Individual	Individual	Couple	Couple	–
Outcomes of concern						
Income or Balance	Income	Income	Income	Income & bequest	Income & bequest	Near-term balance
Horizon	Death of individual	Death of individual	Death of individual	Last surviving	Last surviving	3-years
Preferences						
Attain higher outcomes	Medium	Medium	Medium	Medium	Medium	Low
Avoid lower outcomes	High	High	High	High	Medium	High
Minimum acceptable outcome	Meagre living standard after rent	Meagre living standard after rent	Having to rely on Age Pension	Nil: Age Pension is acceptable	ASFA modest	Substantial loss of capital
Bequest motive	–	–	–	Incidental	Strong	–

(Continued)

Table 1. (Continued)

PANEL B: Utility functions

Type	Reference dependent	Reference dependent	Reference dependent	Reference dependent with bequest term	Reference dependent with bequest term	Power
Reference point	ASFA modest with housing adjustment, single, $42,065	ASFA modest with housing adjustment, single, $42,065	ASFA comfortable, single, $43,200	ASFA modest, couple, $39,666	ASFA comfortable, couple, $60,843	Nil
Parameters – income	*Gains* Curvature: 0.85 *Losses* Curvature: 1.10 Weighting: 1.30	*Gains* Curvature: 0.85 *Losses* Curvature: 1.10 Weighting: 1.30	*Gains* Curvature: 0.85 *Losses* Curvature: 1.10 Weighting: 1.30	*Gains* Curvature: 0.85 *Losses* Curvature: 1.10 Weighting: 1.30	*Gains* Curvature: 0.85 Losses Curvature: 1.05 Weighting: 1.15	Power
Minimum acceptable outcome	Income of $32,000	Income of $32,000	Aged Pension plus $1,000 ($24,824)	–	Income of $39,666	–50% loss of balance
Bequest	–	–	–	*Evaluated as gain:* Curvature: 0.85	Target of $250,000; parameters per income	–

This table presents the attributes and utility functions used to characterise six hypothetical member types. The member attributes appearing in Panel A are chosen by the fund provider, to be used in both segmenting the member base and communicating the type of member for which products are designed. The utility functions presented in Panel B are used to characterise member objectives and preferences over outcomes for the purpose of analysis. Proposed names for the respective products appear in the second row.

a balance of above $500,000. Balance size is not treated as a distinguishing feature for the balance-focused member.

- Homeownership status – The low and medium balance default members are non-homeowners, while the high balance default, the household and the wealthy member are homeowners. Non-homeownership is recognised by adjusting target income, which is based on the retirement standards of the Association of Superannuation Funds of Australia (ASFA) at June 2018.[13] This is done by increasing target income by the difference between estimated rent and the housing costs (excluding water rates) included in the ASFA standard. The rental adjustment is based on median Australian rents for a unit of $356 per week, as reported by CoreLogic[14] for June 2018.
- Excess other assets – The household is assumed to have other assets of below $200,000, while the wealthy member has other assets of above $200,000. This amount may be considered as net of debt, and exclusive of any amounts retained for precautionary or transactional purposes. These assets would be incorporated into the analysis of investment and drawdown strategies, and in determining Age Pension eligibility under the means test.
- Household status – The default members are treated as individuals, while the household and wealthy members are assumed to be couples. Household status impacts income targets, the available Age Pension, horizon (last surviving for couples) and possibly the bequest motive. The balances of the household and wealthy members might be considered as comprising two retirement accounts.

The next two groups of attributes define the outcomes of concern and preferences over those outcomes, respectively. They provide the foundation for utility function specification.

- Outcomes of concern – Default members are assumed concerned with income only; the household and wealthy member with income and bequest; and the balance-focused member with only their balance. The investment horizon over which outcomes are to be evaluated is also nominated. It includes the death of the individual for the default members, the death of the last surviving member for the household and the wealthy member, and the balance after 3 years for the balance-focused member.
- Preferences – The assumed degree of risk or loss aversion is presented as a high/medium/low desire to attain higher outcomes and avoid lower outcomes. Higher outcomes can be conceptualised as above-target under reference-dependent utility and increases in the balance under power utility, with lower outcomes either below-target or a decline in balance. These preferences might be elicited with the assistance of risk profile questionnaires as used by the financial planning industry (see De Ravin et al., 2018). Whether any minimum acceptable outcome exists and its nature is also noted. For example, low and medium balance default members want to avoid a meagre standard of living after accounting for rent, while the wealthy member considers ASFA modest as the lowest acceptable income (noting that they own their home). The balance-focused member wants to avoid a substantial loss of capital. The bequest motive is described as incidental for the household, implying that they value bequests similarly to above-target income. The wealthy member is assumed to have a strong bequest motive.

4.2. Utility function specification

The utility functions selected to represent each member type are summarised in panel B of Table 1. A reference-dependent utility function is chosen for the five member types where concern is over income versus target, coupled with a bequest term for the household and wealthy member. Power utility is chosen for the balance-focused member, who is concerned with the level of their balance without any particular target.

The parameters reported in Table 1 reflect high risk and loss aversion for all but the wealthy member, who is assumed to have medium aversion to losses. The latter is accommodated through lower curvature and weighting functions on losses relative to other members. The reference-dependent utility function parameters are designed to place a proportionately larger penalty on outcomes as they fall further below the target (see discussion in Section 3.2), as reflected in a curvature parameter on losses of greater than one.

Income targets are required under reference-dependent utility. For simplicity, the stated income target is based around the ASFA standard up to age 85 years[15] that is most suitable for the member type. The target for the low balance and medium balance default members is set at the ASFA modest standard adjusted for rent of $42,605. Targets for the high balance default member are set at ASFA comfortable for singles of $42,300, and the household and wealthy members at ASFA modest and ASFA comfortable for couples of $39,666 and $60,843, respectively.

The household bequest motive is treated as incidental by applying no bequest target and evaluating any bequest as above-target income. The wealthy member's strong bequest motive is captured by assuming a target bequest of $250,000 or about four times their income target, while applying the same utility parameters as to income.

Finally, minimum acceptable outcomes are specified for all members except the household, who owns a house and views the Age Pension as acceptable. The low and medium balance default members consider income of below $32,000 as 'unacceptably meagre standard of living', given that they need to pay rent. This notional amount sits around mid-way between their income target and the Age Pension. The high balance default member's minimum acceptable income is set at the Age Pension plus a notional margin of $1000, noting that they do not need to pay rent. The wealthy member considers income of below ASFA modest for couples of $39,666 an unacceptable outcome. The balance-focused member treats 50% loss of capital over 3 years as unacceptable.

Warren (2019) outlines how utility function parameterisation can be guided by examining how outcomes are evaluated across the range. The charts in Figure 1 visualise the selected utility functions. Panels A, B and C plot the reference-dependent utility functions, with the x-axis showing income levels and the y-axis is positioned at the income target. The very large negative utility scores associated with any minimum acceptable outcomes sit well below the lower end of the chart range. The function for the high balance default is similar to the low and medium balance default, except for different income targets and minimum acceptable outcomes. Panel D charts the power utility curve for the balance-focused member, with scores plotted for changes in balance. It typifies the propensity for power utility to heavily penalise lower outcomes under higher risk aversion coefficients. High risk aversion is amplified by imposing a very large negative utility score below the minimum acceptable loss of −50%, with the likely effect of requiring sufficient defensive assets to ensure zero probability of a 50% loss, if possible. Analysis of the gain versus loss utility scores can help reveal the trade-offs made, and is illustrated in Appendix 2.

5. Illustrating the approach: analysis

Analysis of four candidate strategies is presented for the medium balance default (*Income B*), with each strategy involving differing investment mixes and drawdown rules. The analysis is purposefully kept relatively simple. Modelling is conducted over a 30-year horizon, after which the member is assumed to die. The investment strategy involves allocating the balance at retirement between

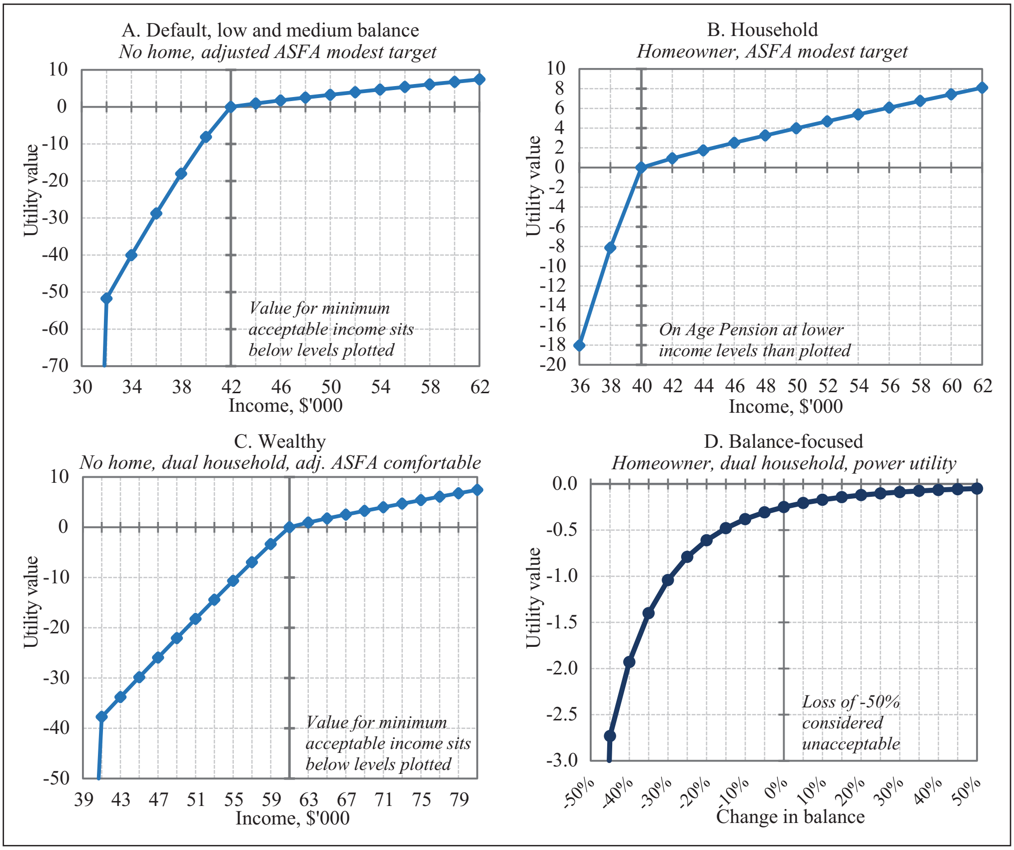

Figure 1. Utility functions for member types.

an account-based pension (ABP) and an annuity, then setting a static growth/defensive allocation within the ABP. Two drawdown strategies are considered. The first (denoted 'draw-to-target') involves drawing a sufficient amount from the ABP to deliver target income after accounting for the annuity and government support, until the ABP is exhausted. The second strictly applies the minimum drawdown rules, to test a familiar and often-followed strategy. The strategies are compared using CEI as discussed in Section 3.2, with selected metrics generated to characterise the outcomes arising from each strategy. Although the modelling is undertaken as if a static strategy is set at retirement and then maintained, it is envisaged that both the investment and drawdown strategies would be reviewed occasionally.

5.1. Modelling assumptions

- Balances at retirement of $300,000, $400,000 and $500,000 are examined.
- Member retires at age 65 years and expects to die at age 95 years, implying a 30-year horizon.
- The reference-dependent utility function appearing in Table 1 is applied, with the income target rounded down to $42,000. A utility value of −100,000,000 is applied below the minimum acceptable income of $32,000.

- The three assets used as building blocks are calibrated as follows:
  - Growth portfolio ('equities') delivers log-normally distributed real returns with an expected mean of 6% per annum[16] (4.7% compound), with a standard error of 2% (to recognise that the expected return is itself a random variable) and a standard deviation of 16%.
  - Defensive portfolio ('bonds') delivers log-normally distributed real returns with a known mean of 1% per annum and a standard deviation of 8%.
  - 'Real' (i.e. inflation protected) 30-year term annuity payments are estimated assuming a real discount rate of 0.5% per annum, indicating 30 yearly payments equal to $37.10 per $1000 invested.
- A management fee of 0.73%–0.74% per annum is applied to the ABP, varying with balance due to a fixed component. Management costs for the annuity are embedded in the discount rate on which the annuity is priced.
- The Age Pension and rental assistance are available, subject to pension eligibility rules under the assets test which is applied with reference to the ABP balance and value attached to the annuity.[17]
- Analysis is conducted in real terms (i.e. all variables adjust with inflation).
- A total of 10,000 yearly simulations of growth and defensive portfolio returns, assuming they are uncorrelated.
- 'Optimised' investment strategies are located by solving for annuity, growth and defensive allocations that maximise expected utility for a given drawdown strategy, with the initial ABP allocation retained for 30 years.
- Scope to switch products is not considered.

5.2. Strategies

Strategy 1: Baseline, optimal – The draw-to-target drawdown strategy is applied, and the asset mix is located that maximises expected utility for this rule. This involves allocating a substantial portion of the balance at retirement to purchasing the annuity, with the residual invested in an ABP with 100% growth. Allocation to the annuity at a $300,000 balance is $125,115 (42% of balance), which suffices to guarantee the minimum acceptable income. Allocation to the annuity at a balance of $400,000 is $180,864 (45%), and at a $500,000 balance is $182,963 (37%). Strategy 1 is used as the baseline against which other strategies are compared.

Strategy 2: Product offered – This is the basis of the product design that will be offered to the retiring member, as later conveyed via Figure 2 and Table 4. The draw-to-target drawdown strategy is applied, and an asset mix imposed. An amount of $125,115 is allocated to the annuity, regardless of balance, being just enough to guarantee the minimum acceptable income. The remainder is placed in the ABP with a traditional 70/30 growth/defensive mix. This asset mix aims to mitigate behavioural and other influences that might undermine acceptance of the product. It recognises that many retirees might prefer to commit less of their balance to the annuity than optimal as it sacrifices flexibility and leaves no residual. This allows the product to be framed as 'purchasing just enough annuity to guarantee a minimum income of $32,000'. It also acknowledges that some retirees might have difficulty accepting a 100% equity mix given its high volatility.

Strategy 3: Minimum drawdown, optimal – The minimum drawdown rules are applied, and the utility-maximising asset mix located for this drawdown strategy. This involves allocating

$125,115 to the annuity at a $300,000 balance, $248,464 (62% of balance) at a $400,000 balance and $229,966 (46%) at a $500,000 balance, with the ABP set at 100% growth. Applying the minimum drawdown rules tends to result in less income being drawn from the ABP earlier in retirement as compared to strategies 1 and 2, especially if returns are weak, with income initially falling short of target in some situations. However, the ABP is never totally exhausted so that some income is drawn throughout and there is always a residual value at age 95 years. Thus, this strategy tends to shift some income that was drawn earlier in retirement under strategies 1 and 2 to either later in the retirement phase or the residual balance (which generates no utility under the setup).

Strategy 4: Product asset mix with minimum drawdown – This strategy applies the minimum drawdown rules along with allocating $125,115 applied to the annuity, with the ABP set at a 70/30 growth/defensive mix.

5.3. Results

The four strategies are initially compared using CEI, and then selected metrics are examined. CEI condenses the distribution of outcomes over 30 years into a single utility-based measure so that strategies can be compared, while indicating the economic gain or loss from choosing one strategy over another. Table 2 reports the CEI estimates.

Focusing first on the $400,000 balance, strategy 1 (baseline, optimal) offers the highest expected utility and hence greatest CEI at $44,142. The other strategies generate lower CEI by −$1430 or −3.2% for strategy 2 (product offered); −$793 or −1.8% for strategy 3 (minimum drawdown, optimal); and −$2337 or −5.3% for strategy 4 (product asset mix, minimum drawdown). A number of insights arise. First, the CEI reductions relative to the baseline optimal strategy suggest that varying the elements of product design results in meaningful but not major loss of utility. Second, the annuity emerges as more effective for managing income risk than including defensive assets in the ABP. This can be seen in that reducing the annuity while increasing defensive exposure within the ABP reduces the CEI by −3.2% for strategy 2 versus strategy 1, and by −3.5% for strategy 4 versus strategy 3. Finally, drawing sufficient income to reach the income target is preferable to applying the minimum drawdown rules, with the latter leading to a CEI reduction of about −2%.

Similar tendencies emerge for $300,000 and $500,000 balances. The notable difference is that the utility losses from adopting sub-optimal strategies are smaller at $300,000 but larger at $500,000. For instance, the total CEI reduction under strategy 4 versus strategy 1 of −7.4% for the $500,000 balance seems quite meaningful. This indicates that strategy design is more important for members with higher than lower balances under this product.

A notable finding is that the utility gains from allocating to the annuity go beyond just securing the minimal acceptable level of income, with expected utility being maximised for balances of $400,000 and $500,000 by allocating more than required to lock in the minimum. This implies that limiting the downside in income is more valuable than holding more growth exposure to seek greater income upside. However, the setup here favours the annuity in various ways. The increasing utility penalty as income falls below target encourages buying a bit more of a guaranteed income stream. The annuity is also made more attractive by placing no value on the residual ABP balance after 30 years, the discount applied on the annuity value under the Age Pension means test and the assumed 8% real return volatility on the defensive portfolio that makes it less attractive as an income hedge.

Table 2 illustrates how utility-based analysis can be used to identify optimal strategies and compare strategies. However, it does not convey any information on the potential outcomes: this is where metrics come in. Table 3 reports selected metrics at age 80, 85, 90 and 95 years. Total

Table 2. Candidate products and certainty equivalent income.

Product	Drawdown strategy	Investment strategy (asset mix)			Certainty equivalent income		
		Annuity		Account-based pension	Suggested treatment		
		$ amount allocated	% of balance		$	Difference $	Difference %
Panel A: $300,000 balance at retirement							
1. Baseline, optimal	Draw-to-target	125,115	42	100% growth	40,180		
2. Product offered	Draw-to-target	125,115	42	70/30	39,837	−344	−0.9
3. Minimum drawdown, optimal	Minimum drawdown rules	125,115	42	100% growth	40,184	4	0.0
4. Product asset mix, minimum drawdown	Minimum drawdown rules	125,115	42	70/30	39,732	−448	−1.1
Panel B: $400,000 balance at retirement							
1. Baseline, optimal	Draw-to-target	180,864	45	100% growth	44,142		
2. Product offered	Draw-to-target	125,115	31	70/30	42,711	−1431	−3.2
3. Minimum drawdown, optimal	Minimum drawdown rules	248,464	62	100% growth	43,349	−793	−1.8
4. Product asset mix, minimum drawdown	Minimum drawdown rules	125,115	31	70/30	41,804	−2337	−5.3
Panel C: $500,000 balance at retirement							
1. Baseline, optimal	Draw-to-target	182,963	37	100% growth	51,815		
2. Product offered	Draw-to-target	125,115	25	70/30	49,387	−2427	−4.7
3. Minimum drawdown, optimal	Minimum drawdown rules	229,966	46	100% growth	51,053	−761	−1.5
4. Product asset mix, minimum drawdown	Minimum drawdown rules	125,115	25	70/30	47,960	−3855	−7.4

This table presents the outputs from the utility-based analysis of four candidate strategies under consideration for medium balance default members. Strategies 1 and 3 involve solving for the allocation to the real term annuity and the static growth/defensive weighting in the account-based pension that maximises expected utility, conditional on the drawdown strategy identified in column 2. Strategies 2 and 4 impose to 70/30 investment strategy in addition to the drawdown strategy. Analysis is based on 10,000 simulated paths over a 30-year horizon and is conducted for balances at retirement of $300,000 (panel A), $400,000 (panel B) and $500,000 (panel C). The columns under the ‘Investment strategy’ heading report allocations to the annuity at retirement and the growth/defensive weights. Certainty equivalent income (CEI) is the certain constant real income stream that delivers the same utility as the uncertain income stream generated by the strategy across 10,000 simulations. Strategies 2, 3 and 4 are compared to strategy 1, which delivers the highest expected utility and hence largest CEI.

Table 3. Selected metrics.

	$ levels					% difference vs. baseline				
Age/Residual at age 95	80	85	90	95	Residual	80	85	90	95	Residual
Target - total income	42,000	42,000	42,000	42,000	0					
Panel A: Income percentiles										
95th percentile										
1. Baseline, optimal	62,679	68,403	79,116	81,986	394,800					
2. Product offered	57,542	60,555	64,199	61,308	237,131	–8%	–11%	–19%	–25%	–40%
3. Minimum drawdown, optimal	58,546	62,955	70,099	70,331	273,076	–7%	–8%	–11%	–14%	–31%
4. Product asset mix, min. drawdown	57,542	60,555	64,227	61,308	237,131	–8%	–11%	–19%	–25%	–40%
75th percentile										
1. Baseline, optimal	51,523	53,083	54,767	52,804	151,575					
2. Product offered	48,974	49,329	49,477	46,939	120,871	–5%	–7%	–10%	–11%	–20%
3. Minimum drawdown, optimal	48,718	49,792	50,964	49,691	106,082	–5%	–6%	–7%	–6%	–30%
4. Product asset mix, min. drawdown	48,992	49,377	49,526	47,055	121,805	–5%	–7%	–10%	–11%	–20%
50th percentile (median)										
1. Baseline, optimal	45,242	44,945	44,771	42,929	71,678					
2. Product offered	44,298	43,628	42,873	42,000	66,810	–2%	–3%	–4%	–2%	–7%
3. Minimum drawdown, optimal	48,718	49,792	50,964	49,691	106,082	8%	11%	14%	16%	48%
4. Product asset mix, min. drawdown	44,394	43,837	43,247	41,209	74,512	–2%	–2%	–3%	–4%	4%
25th percentile										
1. Baseline, optimal	42,000	42,000	42,000	37,047	0					
2. Product offered	42,000	42,000	42,000	39,715	0	0%	0%	0%	7%	nc
3. Minimum drawdown, optimal	41,479	40,999	40,695	39,841	26,387	–1%	–2%	–3%	8%	nc
4. Product asset mix, min. drawdown	40,911	39,904	39,203	37,614	45,419	–3%	–5%	–7%	2%	nc

(Continued)

Table 3. (Continued)

	$ levels					% difference vs. baseline				
Age/Residual at age 95	80	85	90	95	Residual	80	85	90	95	Residual
5th percentile										
1. Baseline, optimal	42,000	34,070	34,070	34,070	0					
2. Product offered	42,000	42,000	32,000	32,000	0	0%	23%	–6%	–6%	nc
3. Minimum drawdown, optimal	39,100	38,653	38,276	37,759	9,541	–7%	13%	12%	11%	nc
4. Product asset mix, min. drawdown	37,550	36,664	35,818	34,726	22,058	–11%	8%	5%	2%	nc
Panel B: Risk metrics										
Probability of shortfall vs. target										
1. Baseline, optimal	2%	8%	16%	26%						
2. Product offered	0%	4%	13%	26%		–1%	–4%	–4%	–1%	
3. Minimum drawdown, optimal	28%	26%	19%	16%		26%	18%	2%	–10%	
4. Minimum drawdown, 70/30 ABP	32%	31%	27%	28%		30%	24%	11%	2%	
Inter-quartile range										
1. Baseline, optimal	9,523	11,083	12,767	15,757	151,575					
2. Product offered	6,974	7,329	7,477	7,224	120,871	–27%	–34%	–41%	–54%	–20%
3. Minimum drawdown, optimal	7,239	8,792	10,269	9,850	79,695	4%	20%	37%	36%	–34%
4. Product asset mix, min. drawdown	8,081	9,473	10,324	9,441	76,386	12%	8%	1%	–4%	–4%

This table reports selected metrics at age 80, 85, 90 and 95 under strategies 1 to 4 as described in Table 2. Percentage differences in the metrics between strategies 2, 3 and 4 and strategy 1 are reported to the right. Total income percentiles reflect the distribution of income defined as the sum of drawdowns from the account-based pension (ABP), annuity payments and government support from the Age Pension plus supplements. Selected percentiles are also reported for any residual balance left over at age 95. Probability of shortfall versus target indicates the likelihood of not achieving the income target at each reported age. Under strategy 1 and 2, this reflects the probability of the ABP being exhausted at that age. Under strategy 3 and 4, shortfall may arise where the minimum drawdown is insufficient to attain the target. The inter-quartile range is the difference between the 75th and 25th percentile, and provides a measure of the spread of outcomes. The best outcome under each metric is shaded in grey. 'nc' denotes not calculable.

income percentiles provide information on the distribution of income defined as the sum of drawdowns from the ABP, annuity payments and government support. Selected percentiles are also reported for the residual balance at age 95 years. Balances not utilised at age 95 years might be interpreted as available to either support income beyond age 95 years or a bequest, and hence may not be totally 'wasted' funds even if not included in the utility calculation. Probability of shortfall captures the likelihood of not achieving the income target at each reported age. Under strategies 1 and 2, this is equivalent to the probability of the ABP being exhausted. Under strategies 3 and 4, failure to achieve target income may occur because the minimum drawdown is insufficient to attain the target: the ABP is never exhausted under these strategies. An inter-quartile range is reported as a measure of the spread of outcomes. This needs to be interpreted carefully, as a high value can reflect upside potential (e.g. possibility of high income) as much as downside risk. The best outputs under each category are highlighted by grey shading.

Table 3 demonstrates how metrics can provide an understanding of the outcomes that various strategies may deliver. Metrics can also be used in communicating the product to members or other stakeholders. For instance, the probability of the ABP being exhausted at various ages forms part of the communication plan presented in Section 6. Metrics also provide a check that a proposed strategy does not inadvertently deliver unacceptable outcomes. Finally, consideration of the range of metrics in Table 3 underscores the difficulty of choosing a preferred strategy based on metrics, with the colour coding indicating that strategy ranking differs across the metrics. This reinforces the advantage of a utility-based approach in allowing strategies to be compared based on a single score, whereas metrics require a stand on which metrics are chosen and how they are traded off.

6. Illustrating the approach: communication

A communication plan is now presented for a retiring member that is offered *MyRetirement Income Fund B* (i.e. the medium balance default fund). The presentation has three components.[18] First is a *letter* (see Figure 2) describing the product that is being recommended to the member, while drawing attention to the availability of other products as well as financial advice. Second is a summary of *product features* (Table 4), including further information on the *attributes* of the member type for which the product is designed and expected *outcomes*. The latter includes expected returns, expected frequency of negative returns over 20 years, and the probability of the ABP being exhausted at selected ages. While the latter is the most relevant measure in the context of the product, return targets and expected frequency of loss are standard measures for Australian superannuation funds. In practice, reported metrics would be tailored to the situation and might be wider in scope. The third component is a *list of alternative products* (Table 5), with emphasis on the type of member each product is designed for. This is a teaser inviting the member to investigate further if another product seems like it might be more suitable.

The product designs and numbers appearing in Figure 2 and Tables 4 and 5 go beyond the modelling of strategy 2 in Section 5, which was based around a predetermined strategy and a fixed 30-year horizon. The presentation assumes that the actual product uses a real life annuity rather than 30-year term annuity, and that the member would be pooled with a cohort of members of similar age as reflected in the name 'MyRetirement Income Fund B 2020–2024'. The intention to review the investment and drawdown strategy every 5 years is also flagged.

The approach relies on meaningful engagement from members to be effective. Here there are grounds for optimism. Both Butt et al. (2018) and Deetlefs et al. (2019) find evidence of a meaningful engagement among a substantial majority of Australian superannuation fund members, with Deetlefs et al. estimating disengaged members at only 22.6%. The need to make a decision

Dear Joseph Smith,

Congratulations on reaching your retirement! As your superannuation fund, we are required to suggest an option for managing your retirement savings going forward. Given your balance of $386,451, our *MyRetirement Income Fund B 2020-24* may be suitable for your needs. This fund has been designed for members who retire at around age 65 between 2020 and 2024, and have the following attributes:

- Balance at retirement of between $300,000 and $500,000;
- Single retiree using their retirement savings to support themselves, with no partner or any other beneficiaries that are relying on being left some money when they die;
- Does not own a home, and has limited other assets, hence is relying on their retirement fund for income;
- Desiring an income of around $42,000 per year (about $808 per week) to live off;
- Would have extreme difficulty living on less than $32,000 per year ($615 per week), allowing for the need to pay rent, meaning that some income supplement is required above the Age Pension and rental assistance.

The key features of *MyRetirement Income Fund B 2020-24* are detailed in Table 5 appearing on the next page. The fund is designed to target an income of $42,000 (adjusted for inflation) by providing a top-up to Government payments including the Aged Pension and rental assistance. The fund aims to deliver the $42,000 income target for as long as possible, after which you will end up receiving an estimated income of around $32,000 for life (adjusted for inflation). This minimum income would come from the Age Pension and rental assistance received from the Government, supplemented by a 'locked-in' income stream arising from an annuity that would be purchased on your behalf.

After purchasing an annuity, the remainder of your balance will be placed in a retirement account that is 70% invested in higher-return and higher-risk assets like equities. Higher-returning assets give a better chance of your funds lasting and supporting your income for longer, but also bring greater exposure to return fluctuations. The outcomes you ultimately receive will depend on movements in the markets and fund performance over time. Good investment returns may mean that you will be able to receive the target income for a longer period of time, and might provide an opportunity to increase the income paid at a later date. Good returns may also result in you being able to leave a bequest, although the fund has not been designed with this in mind. Poor returns may result in you ending up on the minimum income of $32,000 earlier than expected.

You should also give consideration to whether *MyRetirement Income Fund B 2020-24* is appropriate for you. We offer a range of other MyRetirement funds, and services to help you decide if another fund may be more suitable. Table 6 describes our other MyRetirement funds in terms of the type of member for which they are designed. Further information on these funds may be found on our website at *yourfund.com.au/products/*. If you would like further general information on any of our funds, you are welcome to call *YourFund General Advice Services* on (03) 9999 1111, or email us on *general-advice@yourfund.com.au*. You should also consider taking specific personal advice, which will involve an advice fee. If you wish to set up a meeting with one of our financial planners, please call *YourFund Personal Advice Services* on (03) 9999 2222, or email *personal-advice@yourfund.com.au*.

We have attached the Product Disclosure Statement for *the MyRetirement Income Fund B 2020-2*4 for your information, which includes an application form at the back.

Yours sincerely,

Retirement Manager
YourFund

Figure 2. Mock letter to a retiring member.

Table 4. MyRetirement Income Fund B 2020–2024 – features.

Feature	Description				
Designed for member with these attributes	Retiring at around age 65 years between 2020 and 2024 Balance in superannuation fund at retirement of between $300,000 and $500,000 Does not own a home, and hence pays rent Has limited other assets, other than required as a back-up funds in case they are needed Single retiree with no partner Not specifically planning to leave a bequest Desires income of at least $42,000 per year to live off Would have difficulty living off less than $32,000 per year, thus requires a guaranteed income supplement over and above government support including the Age Pension (currently $27,595 per year) and rental assistance (currently $3531 per year)				
Broad approach	Aim is to deliver an income of at least equal to the target of $42,000 per year, increasing with inflation, as long as the balance in the retirement fund lasts Achieved by topping up the Age Pension plus rental assistance Investment made in a life annuity paying an income stream that is sufficient to lock in minimum total income of $32,000 per year, increasing with inflation Remainder of funds placed in a retirement account (account-based pension), which is substantially invested in growth assets to increase the chance of generating the $42,000 income target for longer				
Payments	Life annuity pays income of $4650 per year, with this amount increasing with inflation (measured by the Consumer Price Index), until death Income distributed from the retirement account is equal to either: Difference between $42,000 per year, increasing with inflation, and the amount received from the Age Pension, rental assistance and the annuity Amount required under the government's minimum drawdown rules, if greater Larger drawdowns possible on request, but will reduce the amount available to support future income				
Left-over balance	Paid to nominated beneficiaries upon death				
Fees	Investment fee of 0.50% per year Administration fee of 0.20% per year, plus $2.50 per week ($130 per year) Equates to total fee of 0.73% per year at a balance of $400,000				
Asset allocation	$125,000 is invested in a life annuity, paying $4650 per year, increasing with inflation, until death occurs. This locks in the minimum income target of $32,000 Residual invested in a retirement account with 70% in growth assets (e.g. equities, property), and 30% in defensive assets (e.g. bonds, cash)				
Potential outcomes for the retirement account (note: excludes the annuity)	Expected long-run return, after fees: 6.5% per year (4.5% after inflation) Frequency of negative returns: 6 of 20 years *Likelihood of the retirement account lasting to various ages for three starting balances at age 65 years*:				
	Probability of balance lasting to:	80 years	85 years	90 years	95 years
	For starting balance of $300,000	92%	70%	50%	36%
	For starting balance of $400,000	99.7%	96%	87%	74%
	For starting balance of $500,000	99.9%	99.5%	96%	89%
Advice	General advice on other *YourFund* products – available at no charge Specific personal advice – fee will apply				
Ongoing review	Fund investment strategy will be reviewed on a 5-year cycle, to consider whether the asset mix in the retirement account should be changed, or the income target might be safely increased Members are invited to occasionally review whether the fund remains suitable through taking general advice or specific individual advice				

This table summarises the features of the MyRetirement product that is offered to the medium balance default member. It is presented to the member as a 'factsheet' in conjunction with the letter appearing as Figure 2, a description of other options as per Table 5 and the product disclosure statement.

Table 5. Other MyRetirement fund options offered by *YourFund*..

Fund	MyRetirement Income Fund C 2020–2024	MyRetirement Family Fund, 2020–2024	MyRetirement Well-off Fund, 2020–2024	MyRetirement Nest-egg Fund, 2020–2024
Type of member designed for	– Balance at retirement exceeding \$500,000 – Homeowner – Limited other assets – Single retiree – Leaving a bequest not considered necessary – Desired income about \$43,000 per year – Keen to avoid relying on the Age Pension	– Balance at retirement of \$200,000–\$500,000 – Homeowner – Meaningful other assets (e.g. up to \$200,000) but not overly wealthy – Has a partner – Leaving a bequest would be valued, but not essential – Desired income about \$40,000 per year – Can live on the Age Pension if required	– In excess of \$700,000 in wealth in addition to the family home, including balance at retirement plus any other assets – Homeowner – Has a partner – Keen to leave a substantial bequest – Desired income about \$60,000 per year – Living on income of below \$40,000 would be quite difficult	– Focused on protecting retirement account balance over the short-medium term, say 3 years – Possibly anticipating using the balance to support a spending need in foreseeable future – This fund might be combined with another fund that is focused on generating income

This table summarises the key features of other MyRetirement products being offered by the provider, focusing on the attributes of the type of member for which each product is designed. It is presented to the member as information to help them decide if another product option might be more suitable for them, so they can take appropriate action if they wish. It is provided in conjunction with the letter appearing as Figure 2, the product features 'factsheet' as per Table 4 and the product disclosure statement.

at retirement makes it likely that attention levels and willingness to engage will be relatively high at that point. The challenge is to convert engagement into constructive action by communicating the product and the alternative options in a way that members can comprehend and hence make appropriate choices. The communication plan presented here is a preliminary suggestion to achieve this, but undoubtedly leaves room for improvement and would require member testing in any event.

7. Conclusion

An approach is proposed for designing a menu of retirement products – or CIPRs – where utility functions provide the primary mechanism for evaluating the strategies underpinning product design. The approach is illustrated using a mock menu of Australian MyRetirement products. The illustrative example demonstrates the process of identifying and characterising member types for which products will be designed; the tailoring of utility functions to represent each member type; the identification of a preferred investment and drawdown strategy by applying utility-based analysis in conjunction with metrics; and the communication of products to a retiring member. It shows how utility functions can be an effective analysis tool, without the need to mention them when engaging with investors or other stakeholders. Rather, communication can be focused around attributes that describe the type of investor for which a product is designed, the product features and the outcomes that it might deliver. In summary, the approach applies utility functions as the primary engine of analysis operating 'under the hood', while engagement with members occurs in terms that they can hopefully understand.

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Notes

1. See https://treasury.gov.au/programs-and-initiatives-superannuation/retirement-framework.
2. https://theconexusinstitute.org.au/resources/members-default-utility-function-mduf/
3. See https://www.legislation.gov.au/Details/C2019A00050.
4. Impact of precautionary and transactional assets on Age Pension eligibility should be taken into account.
5. Prospect theory also entails a preliminary editing stage where prospects are reduced to a subset for evaluation, as well as application of decision weights that transform the probabilities attached to outcomes. These elements make prospect theory a theory of non-expected utility.
6. The need to provide for aged care is more relevant for members who do not own a house, or are wealthy and anticipate receiving limited government aged care support, see https://www.myagedcare.gov.au/aged-care-home-costs-and-fees.
7. The illustrative analysis presented in this article uses this approach.
8. The Association of Superannuation Funds of Australia (ASFA) Retirement Standards are an application of the consumption budget approach for Australia, see https://www.superannuation.asn.au/resources/retirement-standard.
9. The Australian Government Actuary estimates for Australians may be found at http://www.aga.gov.au/publications/life_table_2015-17/default.asp.
10. The method is straightforward where expected utility is estimated over a known horizon (T) without mortality weighting. For power utility, $CEC_{PU} = \{(1-\rho)\, E[U_{PU}] \,/\, T\}$^$(1/(1-\rho))$.
    For reference-dependent utility: $CEC_{RD|U>0} = C^* + \{E[U_{RDU}] \,/\, (T)\}$^$(1/\gamma_1)$ and,
    $CEC_{RD|U<0} = C^* - \{-E[U_{RDU}] \,/\, (\lambda T)\}$^$(1 \,/\, \gamma_2)$.
    See Appendix 1 for definition of terms.
11. See Australian Securities and Investments Commission (ASIC) Consultation paper 332, see https://download.asic.gov.au/media/5853864/cp332-published-17-november-2020.pdf
12. The Treasury's Retirement Income Covenant Position Paper (May 2018, p. 5) states the following: 'The Government would require trustees to provide guidance (which may or may not be financial advice) or intra-fund advice tools to help members navigate between the retirement income products offered by the fund. The tools should assist members in making the most appropriate choice, given their needs and preferences'. See https://treasury.gov.au/consultation/c2018-t285219.

13. https://www.superannuation.asn.au/ArticleDocuments/269/ASFA-RetirementStandard-Budgets-Jun2018.pdf.aspx?Embed=Y.
14. https://www.corelogic.com.au/reports/quarterly-rental-review.
15. This avoids the need to quote multiple target numbers in the analysis, which is intended as an illustration.
16. In terms of compound returns, this equates to a long-term equity risk premium of about 4% per annum.
17. Under the current rules (see https://www.humanservices.gov.au/individuals/topics/income-streams/27671), 60% of the annuity purchase value is incorporated into the assets tests until age 84 years, and 30% incorporated from age 85 years.
18. The presentation assumes that safe harbour provisions are brought into play, such that describing the recommended product and other available options in manner displayed does not fall foul of the rules around provision of financial advice.
19. Non-time-separable utility functions such as Epstein and Zin (1989) and Weil (1989) require recursive estimation.

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Appendix 1

Utility functions

This Appendix provides equations for the utility functions referred to in this article, and discusses some technical aspects of their application.

A1.1. Reference-dependent utility. The reference-dependent utility function is described by equation (1)

$$U(C)_{RDU,t} = \mathbf{1}_{(C_t > C_t^*)}\left(C_t - C_t^*\right)^{\gamma_1} - \mathbf{1}_{(C_t < C_t^*)}\left(\lambda\left(C_t - C_t^*\right)^{\gamma_2}\right) + \mathbf{1}_{(C_t = C_t^*)}0 \qquad (1)$$

where $U(C)_{RDU}$ is the reference-dependent utility; C is the consumption; C^* is the target consumption; $\mathbf{1}$ is an indicator function that equals 1 when the condition is satisfied and 0 otherwise; γ_1 is the curvature parameter when consumption exceeds the target; γ_2 is the curvature parameter when consumption is below the target; λ is the weighting on losses (i.e. loss aversion parameter); and t is the time period.

A1.2. Power utility. The power utility function is described by equation (2)

$$U(C)_{PU,t} = \frac{C_t^{(1-\rho)}}{1-\rho} \quad \left(\text{If } \rho = 1, U_{PU} = ln(C), \text{i.e. log utility}\right) \qquad (2)$$

where $U(C)_{PU}$ is the power utility and ρ is the coefficient of relative risk aversion.

A1.3. Utility with consumption floor. The commonly used utility function with a consumption floor is described by equation (3)

$$U(C)_{CF,t} = \frac{\left(C_t - C\right)^{(1-\rho)}}{1-\rho} \quad \left(\text{If } \rho = 1,\ U_{CF} = ln\left(C - C\right)\right) \qquad (3)$$

where $U(C)_{CF}$ is the utility with a consumption floor and C is the consumption floor. This function is similar to power utility but with the floor adding 'leverage' such that curvature increases as consumption approaches the floor. Care needs to be taken as utility is undefined below the floor.

A1.4. Bequests. Equations (4) and (5) follow Lockwood (2018). They reflect a function that generates utility from bequests that is commonly used in conjunction with power utility

$$U(B)_{i,t} = b\frac{\left(\left(\frac{\phi}{1-\phi}\right)C_b + B_{i,t}\right)^{1-\rho}}{1-\rho} \tag{4}$$

$$b = \left(\frac{\phi}{1-\phi}\right)^{\rho} \tag{5}$$

where $U(B)$ is the utility generated by a bequest; B is the bequest; $C_b \geqslant 0$ is a threshold consumption level above which the preference for bequests applies; and $\phi \in [0, 1]$ controls the strength of the bequest motive. $\phi = 0.5$ implies that bequests are valued equally to consumption, with the bequest motive strengthening as ϕ increases towards 1. The intuition is that the bequest term is included in the utility function to generate a penalty for failing to leave a bequest, which is implicitly traded off against the consumption stream. To help understand the relative weights placed on bequests and consumption, Cocco et al. (2005) assume that $C_b = 0$ and set b directly, then make the following comment: 'Loosely speaking, b can be interpreted as the number of years of consumption of his descendants that the investor wants to save for, or the number of years by which the investor's horizon is effectively increased' (p. 516). For instance, $\phi = 0.65$ and $\rho = 3$ would deliver $b = 6.4$, that is, around 6 years. Assuming $C_b > 0$ ensures that the trade-off between consumption and bequests does not cut in until consumption has reached this threshold, with no penalty for failing to leave a bequest if $C \leqslant C_b$. In the absence of imposing a value for C_b, the penalty for leaving a small bequest may become quite large.

The above formulation for power utility provides clues on how to incorporate a bequest motive into reference-dependent utility. A key issue is whether bequests should be treated as an incidental gain versus an outcome over which there is a strong preference. Bequests may be treated as an incidental gain by setting a target bequest of $B^* = 0$, in which case any bequest will be evaluating using the curvature for gains with no penalty for failing to leave a bequest. A strong bequest preference may be imposed by setting a bequest target of $B^* = x > 0$, thus evaluating any bequest less than x as a loss.

A1.5. Estimating expected utility. Equation (6) describes how expected utility is estimated by aggregating utility scores across the distribution of outcomes. This equation is common in the literature (e.g. Cocco et al., 2005; Lockwood, 2018), and represents the calculation for a time-separable utility function.[19] It describes how utility from each period is added to generate the total utility for each path i, with expected utility estimated by averaging across N paths

$$E_0[U] = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{t=1}^{T}\delta^t\left(\prod_{t=1}^{t-1}p_t\right)\left\{p_t U(C)_{i,t} + (1-p_t)U(B)_{i,t}\right\}\right] \tag{6}$$

where δ is the time preference parameter; p_t is the probability of survival over period t; $U(C)_{i,t}$ and $U(B)_{i,t}$ are respectively the utility from consumption (or income) and bequests, arising in path i during the period t; N is the number of paths; and T is the horizon. Equation (6) provides for time preference and probability of survival.

Appendix 2

Evaluation of utility function parameters

This Appendix demonstrates how utility functions may be evaluated by examining the utility scores placed on outcomes, using the functions proposed in Table 1 as an example. Table 6 presents the scores generated by selected functions, initially focusing on gains and losses separately and then reporting the ratio of absolute scores on gains versus losses of an equivalent magnitude. Gains and losses are defined relative to the income target under reference-dependent utility, and the change in balance under power utility. The minimum acceptable outcome at which an arbitrarily large negative utility score is applied is also indicated. Panel A reveals that the utility function for the default members places scores of gains that are 11%–17% of those applied to equivalent losses. The corresponding ratio for the wealthy member is 20%–28% (see panel B), meaning that they are assumed to be relatively more willing to accept the risk of loss in the pursuit of gain. To place these ratios in context, Warren (2019) shows how the parameters of Tversky and Kahneman (1992) imply the scores applied to gains are 0.444 of those for losses ($= 1/\lambda = 1/2.25$), whereas the parameters assumed by Blake et al. (2013) imply that gains are valued at less than 0.01 of equivalent losses. Under power utility (see panel C), the ratios are consistent with a reasonable level of tolerance for smaller reductions in balance, for example, the utility score on a 10% increase is 0.45 of the utility score placed on a 10% decline in balance. The aversion to lower balances increases markedly as the reduction in balance becomes larger in magnitude. For example, the score associated with a 30% gain in balance is only 0.08 of the magnitude of the score associated with a 30% loss. This method for analysing utility functions can assist in deciding whether the implied trade-offs are appropriate for the member type in question.

Table 6. Utility scores placed on gains and losses.

Panel A: Default member – low and medium balance

Gains			Losses			Gain vs. loss
Income vs. target	Utility score	Discount on gain	Income vs. target	Utility score	Penalty on loss	|Gain score| / |Loss score|
$2000	$935	0.47	–$2000	–$5560	2.78	0.17
$4000	$1745	0.44	–$4000	–$11,918	2.98	0.15
$6000	$2514	0.42	–$6000	–$18,617	3.10	0.14
$8000	$3257	0.41	–$8000	–$25,547	3.19	0.13
$10,000	$3981	0.40	–$10,000	–$32,655	3.27	0.12
$12,000	$4691	0.39	<–$10,065	Large −ve	$\rightarrow \infty$	$\rightarrow 0$
$14,000	$5389	0.38				
$16,000	$6077	0.38				
$18,000	$6757	0.38				
$20,000	$7429	0.37				

(Continued)

Table 6. (Continued)

Panel B: Wealthy member

Gains			Losses			Gain vs. loss
Income vs. target	Utility score	Discount on gain	Income vs. target	Utility score	Penalty on loss	|Gain score| / |Loss score|
$2000	$935	0.47	–$2000	–$3363	1.68	0.28
$4000	$1745	0.44	–$4000	–$6964	1.74	0.25
$6000	$2514	0.42	–$6000	–$10,660	1.78	0.24
$8000	$3257	0.41	–$8000	–$14,419	1.80	0.23
$10,000	$3981	0.40	–$10,000	–$18,226	1.82	0.22
$12,000	$4691	0.39	–$12,000	–$22,072	1.84	0.21
$14,000	$5389	0.38	–$14,000	–$25,950	1.85	0.21
$16,000	$6077	0.38	–$16,000	–$29,855	1.87	0.20
$18,000	$6757	0.38	–$18,000	–$33,786	1.88	0.20
$20,000	$7429	0.37	–$20,000	–$37,738	1.89	0.20
$22,000	$8094	0.37	–$21,177	Large −ve	$\rightarrow \infty$	$\rightarrow 0$

Panel C: Balance-focused member

Gains		Losses		Gain vs. loss
3-year change in balance	Utility score	3-year change in balance	Utility score	|Gain score| / |Loss score|
5%	−0.206	–5%	−0.307	0.67
10%	−0.171	–10%	−0.381	0.45
15%	−0.143	–15%	−0.479	0.30
20%	−0.121	–20%	−0.610	0.20
25%	−0.102	–25%	−0.790	0.13
30%	−0.088	–30%	−1.041	0.08
35%	−0.075	–35%	−1.401	0.05
40%	−0.065	–40%	−1.929	0.03
45%	−0.057	–45%	−2.732	0.02
50%	−0.049	–50%	Large −ve	$\rightarrow 0$

This table analyses the utility scores arising under the utility functions proposed for the low and medium balance default member (panel A), the wealthy member (panel B) and the balance-focused member (panel C), as described in Table 1. The analysis focuses on the scores placed on gains and losses, defined relative to target income under reference-dependent utility functions in panels A and B, and as changes in wealth under the power utility function in panel C. The final column reports the ratio of the absolute utility scores on gains versus losses of equal magnitude. These estimates indicate how each utility function trades off gains versus losses by revealing the extent to which gains are discounted relative to losses of the same magnitude. The third and sixth columns in panels A and B report the discount on gains and the penalty (i.e. multiplier) on losses under the reference-dependent utility functions where scores may be interpreted as a scaling of gains and losses, respectively.